Study of Long Distance Contributions to $K \to n\pi\nu\bar{\nu}$

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Abstract

We calculate long distance contributions to $K \to \pi\nu\bar{\nu}$, $\pi\pi\nu\bar{\nu}$, and $\pi\pi\pi\nu\bar{\nu}$ modes within the framework of chiral perturbation theory. We find that these contributions to decay rates of $K \to \pi\nu\bar{\nu}$ and $K \to \pi\pi\nu\bar{\nu}$ in the chiral logarithmic approximation are at least seven orders of magnitude suppressed relative to those from the short distance parts. The long distance effects in this class of decays are therefore negligible.

13.20Eb, 12.39Fe, and 11.30Rd

I. INTRODUCTION

The K decays receive contributions from both long and short distance effect. The long distance one is the sum of all sorts of nonperturbative effects such as hadron formation, symmetry breaking, *etc.* These effects realize themselves as the coefficients in the chiral Lagrangian and can only be determined by experimental fits. The short distance contribution accounts for the perturbative effect of the underlying standard model dynamics, in which the amplitude can be calculated explicitly with the aid of meson decay constants in the hadron matrix elements. In principle, we could directly test the standard model parameters if the short distance effect in the decay is able to be extracted from the total amplitude, provided the long distance effect can also be calculated separately. However, the long distance effects in many K decays dominate the decay amplitudes and thus the short distance contributions are buried under the overwhelming long distance backgrounds [1]. In such cases, it would be difficult to learn the physics of the standard model.

Fortunately, there exists a class of K decay modes such as $K^+ \to \pi^+\nu\bar{\nu}$, dominated by the short distance effects [2–5]. These decays have been playing important roles for us to understand the structure of weak interaction with high precision. In general, they are suppressed by GIM mechanism and the leading short distance contributions arise from one-loop diagrams, resulting in that the decay amplitudes involve the CKM matrix elements and heavy quark masses such as m_c and m_t [6,7]. Problems concerning the hadron matrix element persist but can be better managed. The physics that how the quarks form hadrons, namely the nonperturbative effect, is lumped into the measured constants f_π and f_K. The effect of the short distance contribution can be calculated within the framework of the standard model and the result is factorized. Since the decay amplitudes depend explicitly on various weak interaction parameters such as V_{td} or m_t, they can be used to extract these parameters from experimental data. A common practice, for example, is to plot the decay rate as a function of m_t to give us some insights on the top quark mass, which, in turn, could eliminate the uncertainty in the determination of the CKM parameters. It is clear that the

result relies crucially upon the domination of the short distance contribution to the decay rate. In principle, the long distance effect may also contribute to the decay amplitude at the same order of magnitude. There is no apparent reason why the effect should be suppressed. In this paper, however, we will give an explicit calculation in chiral perturbation theory to illustrate that such effect in the class of $K \to n\pi\nu\bar{\nu}$ decays is negligible compared with the short distance one, where n stands for the number of π mesons. Especially we will show that the long distance parts of the decay rates for the processes $K \to \pi\nu\bar{\nu}$ and $K \to \pi\pi\nu\bar{\nu}$ are at least seven orders of magnitude smaller than those from the short distance contributions in the chiral logarithmic approximation. The origin for the suppression is twofold. The long distance contribution, which is believed to arise mainly from u quark loop in the underlying theory, is much smaller compared with the short distance contribution which contains heavy quark loop contribution [8]. On the other hand, all the long distance contributions in $K \to n\pi\nu\bar{\nu}$ modes, calculated within the framework of chiral perturbation theory, start to receive contributions at $O(P^4)$ as a result of incompatibility of Lorentz invariance and gauge invariance. The dominance of short distance contribution ensures the validity of relating the decay rate of this sort of processes to the weak interaction parameters.

The article is organized as follows. In Sec. II, we construct the Lagrangian which is relevant to the decays of interest. In Sec. III, we calculate the decay amplitudes of $K \to n\pi\nu\bar{\nu}$ and give numerical evaluations. The results of the short and long distance contributions to the decay rates are compared. We conclude in Sec. IV.

II. CHIRAL LAGRANGIAN INVOLVING Z^0

The processes $K \to n\pi\nu\bar{\nu}$ are mainly mediated by $K \to n\pi Z^0$ and followed by $Z^0 \to \nu\bar{\nu}$. We restrict our discussion on the Z^0 mediated diagrams only. We will not consider the box diagram with two external W bosons as intermediate states. The couplings of $Kn\pi Z^0$ are the major concern of this section. The effect of Z^0 can be incorporated into the chiral Lagrangian of mesons by treating it as an external gauge field. There are four pieces in

the Lagrangian, which are relevant to the analysis of $Kn\pi Z^0$ couplings, corresponding to the strong interaction, $\mathcal{L}_2$, the weak interaction, $\mathcal{L}_2^{\Delta S=1}$, the Wess-Zumino-Witten (W.Z.W.) anomaly [9,10], $\mathcal{L}_{WZW}$, and the weak anomaly [11], $\mathcal{L}_A^{\Delta S=1}$, respectively. The first two are of $O(P^2)$ while the last two of $O(P^4)$ in the chiral power counting. In order to consider $K \to n\pi\nu\bar{\nu}$ to $O(P^4)$ consistently, the $O(P^4)$ counter terms of strong and weak interactions, as well as the loop corrections, should be all included in the analysis. However, as is well known, the counter terms contain numerous unknown coefficients [12] and it makes the calculation impractical. The strategy adopted in the present work is first to decide whether the $O(P^2)$ Lagrangian contributes to the processes or not. If it does, the amplitude can be calculated explicitly without the uncertainty arising from the unknown coefficients and the analysis will be terminated there since the $O(P^2)$ Lagrangian yields the dominant contribution. If $O(P^2)$ terms in the Lagrangian do not contribute, then the long distance contribution is further suppressed. In principle, three kinds of contributions may enter the $O(P^4)$ analysis, which are loop corrections, counter terms and anomalies, respectively. But, as will be shown in the numerical analysis, the long distance effect in the decay branching ratio is at least relatively 10^{-7} smaller than the short distance effect [5,13], so that exact numerical evaluations are not necessary. It is sufficient to use the anomalies or the loop corrections to estimate the orders of magnitude of the long distance effect.

In the $O(P^2)$ Lagrangian, Z^0 is incorporated into the Lagrangian by gauging derivatives. The Z^0 coupling, with the mixing of the hypercharge, contains both left and right handed currents. The right handed current has only octet coupling while the left one gets both octet and singlet ones [4,5,13]. The $U(1)$ symmetry, resulting in the singlet coupling, is not the part of the chiral symmetry. We include it by first assuming nonet symmetry and then use a parameter ξ to indicate the degree of the nonet symmetry breaking with $\xi = 1$ in the nonet symmetry limit. The covariant derivative reads as

$$\begin{aligned} D_\mu U &= \partial_\mu U - ir_\mu U + iUl_\mu \\ &\equiv \partial_\mu U + \frac{ig}{cos\theta_W}(UQ - \frac{\xi}{6}UI - sin^2\theta_W\,[U, Q])Z^0_\mu, \end{aligned} \tag{1}$$

where Q is the quark charge matrix, $Q = diag(2/3, -1/3, -1/3)$, and U is the nonlinear realization of meson octet

$$U = exp(i\Phi/f_\pi), \tag{2}$$

with

$$\Phi = \phi^a\lambda^a = \sqrt{2}\begin{pmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix}. \tag{3}$$

and $f_\pi = 93MeV$ being the pion decay constant. By this identification of the covariant derivative, the $O(P^2)$ strong interaction Lagrangian is given by

$$\mathcal{L}_2 = \frac{f_\pi^2}{4}Tr\left[D_\mu U^\dagger D^\mu U + 2B_0 M(U + U^\dagger)\right], \tag{4}$$

where $M = diag(m_u, m_d, m_s)$ is the quark mass matrix. Note that the covariant derivative defined here is different from that in Ref. [4]. In fact, the left and right handed currents were switched in Ref. [4]. It is not consistent with the required chiral transformation property [13]. The coefficient of the mass term is determined by the ratios of meson and quark masses

$$B_0 = \frac{m_K^2}{m_u + m_s} = \frac{m_\pi^2}{m_u + m_d} = \frac{3m_\eta^2}{m_u + m_d + 4m_s}. \tag{5}$$

The $O(P^2)$ weak interaction Lagrangian is given by

$$\mathcal{L}_2^{\Delta S=1} = G_8 f_\pi^4 Tr\lambda_6 D_\mu U^\dagger D^\mu U, \tag{6}$$

with $G_8 \approx 9.1 \times 10^{-6} GeV^{-2}$ determined from $K \to \pi\pi$. The relevant part of W.Z.W. anomaly has only one gauge boson coupling

$$\mathcal{L}_{WZW} = -\frac{1}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}Tr(\Sigma_\mu^L\Sigma_\nu^L\Sigma_\alpha^L l_\beta - \Sigma_\mu^R\Sigma_\nu^R\Sigma_\alpha^R r_\beta), \tag{7}$$

where $\Sigma_\mu^L = U^\dagger\partial_\mu U$ and $\Sigma_\mu^R = U\partial_\mu U^\dagger$. The direct weak anomaly [11] contains four unknown parameters a_i $(i = 1\cdots 4)$ and, explicitly, one has,

$$
\mathcal{L}_A^{\Delta S=1} = \frac{G_8 f_\pi^2}{16\pi^2}\Big\{2a_1 i\varepsilon^{\mu\nu\alpha\beta} Tr\lambda_6 L_\mu Tr L_\nu L_\alpha L_\beta
$$

$$
+a_2 Tr\lambda_6[U^\dagger \tilde{F}_R^{\mu\nu}U, L_\mu L_\nu] + 3a_3 Tr\lambda_6 L_\mu Tr(\tilde{F}_L^{\mu\nu} + U^\dagger \tilde{F}_R^{\mu\nu}U)L_\nu
$$

$$
+a_4 Tr\lambda_6 L_\mu Tr(\tilde{F}_L^{\mu\nu} - U^\dagger \tilde{F}_R^{\mu\nu}U)L_\nu\Big\}\,. \tag{8}
$$

These unknown parameters are believed to be of order one. The fields in Eq. (8) are defined as

$$
\begin{aligned}
&L_\mu = iU^\dagger D_\mu U\ , R_\mu = iUD_\mu U^\dagger \\
&F^L_{\mu\nu} = \partial_\mu l_\nu - \partial_\nu l_\mu\ , F^R_{\mu\nu} = \partial_\mu r_\nu - \partial_\nu r_\mu \\
&\tilde{F}^{\mu\nu}_{L,R} = \varepsilon^{\mu\nu\rho\sigma} F^{L,R}_{\rho\sigma}.
\end{aligned} \tag{9}
$$

Since K could mix with π through a weak transition, it is cumbersome to use the octet fields defined in Eq. (3) to calculate the amplitudes. The new basis which simultaneously diagonalizes $\mathcal{L}_2$ and $\mathcal{L}_2^{\Delta S=1}$ [14] is given by

$$
\begin{aligned}
\pi^+ &\to \pi^+ - \frac{2m_K^2 f_\pi^2 G_8}{m_K^2 - m_\pi^2} K^+ \\
K^+ &\to K^+ + \frac{2m_\pi^2 f_\pi^2 G_8^*}{m_K^2 - m_\pi^2}\pi^+ \\
\pi^0 &\to \pi^0 + \frac{\sqrt{2} m_K^2 f_\pi^2}{m_K^2 - m_\pi^2}(G_8 K^0 + G_8^* \bar{K}^0) \\
K^0 &\to K^0 - \frac{\sqrt{2} m_\pi^2 f_\pi^2 G_8^*}{m_K^2 - m_\pi^2}\pi^0 + \sqrt{\frac{2}{3}}\frac{m_\eta^2 f_\pi^2 G_8^*}{m_\eta^2 - m_K^2}\eta \\
\eta &\to \eta - \sqrt{\frac{2}{3}}\frac{m_K^2 f_\pi^2}{m_\eta^2 - m_K^2}(G_8 K^0 + G_8^* \bar{K}^0)\,.
\end{aligned} \tag{10}
$$

In this transformed basis, the vertices $K^+\pi^+$, $K_L\pi^0$, $K_L\pi^0 Z^0$ and $K^+\pi^+ Z^0$ are eliminated and the numbers of Feynman diagrams for the processes of interest are reduced substantially.

III. AMPLITUDES AND BRANCHING RATIOS

The phase space allowed modes for K decaying to $n\pi\nu\bar{\nu}$ are $\pi\nu\bar{\nu}$, $\pi\pi\nu\bar{\nu}$ and $\pi\pi\pi\nu\bar{\nu}$, i.e., $n \le 3$. It is easily seen that processes involving only neutral particles receive no contribution

from the Lagrangian. The long distance contributions in $K_L \to \pi^0\nu\bar{\nu}$, $\pi^0\pi^0\nu\bar{\nu}$ and $\pi^0\pi^0\pi^0\nu\bar{\nu}$ are therefore trivial and we shall not include these modes in the remaining discussions of the paper. The amplitudes and branching ratios of the rest decays of $K \to n\pi\nu\bar{\nu}$ are analyzed with the aforementioned strategy as follows.

$K^+ \to \pi^+\nu\bar{\nu}$

The long distance contribution of this mode has recently been studied in various approaches [2,4,13]. It is found that neither the $O(P^2)$ Lagrangian nor the anomaly Lagrangian contributes to this process [13]. So the amplitude is at most of $O(P^4)$. The loop contribution [13] is given by

$$A(K^+ \to \pi^+\nu\bar{\nu}) = -\frac{i\alpha G_8(1-2\sin^2\theta_W)}{64\pi M_Z^2 \sin^2\theta_W \cos^2\theta_W} J(m_K^2)(P_K+P_\pi)^\mu \bar{\nu}\gamma_\mu(1-\gamma_5)\nu, \tag{11}$$

where the loop function $J(m^2)$ is defined as

$$\begin{aligned} J(m^2) &= \frac{1}{i\pi^2}\int d^n q \frac{1}{q^2-m^2} \\ &= m^2(\Delta - \ln\frac{m^2}{4\pi^2 f_\pi^2}). \end{aligned} \tag{12}$$

The divergent part in Eq. (12) is given by

$$\Delta = \frac{2}{\epsilon} - \gamma - \ln\pi + 1, \tag{13}$$

where γ is the Euler number and $\epsilon = 4-n$. The decay rate can be evaluated analytically and it is found to be

$$\begin{aligned} \Gamma(K^+ \to \pi^+\nu\bar{\nu}) = &\frac{\alpha^2 G_8^2 m_K^5 (1-2\sin^2\theta_W)^2}{2^{19}\pi^5 M_Z^4 \sin^4\theta_W \cos^4\theta_W} \\ &\cdot(1-8r_\pi+8r_\pi^3-r_\pi^4-12r_\pi^2\ln r_\pi)|J(m_K^2)|^2, \end{aligned} \tag{14}$$

with $r_\pi = m_\pi^2/m_K^2$. The long distance contribution gives rise to the branching ratio

$$Br(K^+ \to \pi^+\nu\bar{\nu}) \sim 7.7 \cdot 10^{-18}\,, \tag{15}$$

which is roughly 10^{-7} smaller than that of the short distance contribution [3,6]. We note that our result in Eq. (15) is different from that of Refs. [2] and [4]. Although our work is analyzed within the same framework, namely chiral perturbation theory as in [4], we find that the tree level amplitude of $K^+\pi^+Z^0$ vanishes identically as shown in Sec. II, which is only true in the limit of the large N_c in [4]. This difference may arise from the different identification of left-handed and right-handed currents [13]. However, we could not be able to find the reason of the discrepancy with Ref. [2]. Further study on this issue is needed.

$$K_L \to \pi^+\pi^-\nu\bar{\nu}$$

This mode receives no contribution from the $O(P^2)$ Lagrangian. The amplitudes arising from the anomaly Lagrangian can be written as

$$\begin{aligned} A(K_L \to \pi^+\pi^-\nu\bar{\nu}) = &-\frac{i\alpha G_8}{4\pi f_\pi M_Z^2 \sin^2\theta_W \cos^2\theta_W}\bar{\nu}\gamma_\mu(1-\gamma_5)\nu \\ &\cdot\Big[6a_1 - 6a_3 - 2a_4 + 2\sin^2\theta_W(a_2+2a_4) - \xi\Big]\,\varepsilon^{\mu\nu\alpha\beta}P_{K\nu}P_{\pi^+\alpha}P_{\pi^-\beta}. \end{aligned} \quad (16)$$

The corresponding differential decay rate can be evaluated analytically

$$\begin{aligned} \frac{d^3\Gamma}{ds_\pi ds_\nu d\cos\theta_\pi} = &\frac{\alpha^2 G_8^2 \sigma_\pi^3 X^3 \sin^2\theta_\pi s_\pi s_\nu}{2^{15}\pi^7 f_\pi^2 M_Z^4 m_K^3 \sin^4\theta_W \cos^4\theta_W} \\ &\cdot\Big[6a_1 - 6a_3 - 2a_4 + 2\sin^2\theta_W(a_2+2a_4) - \xi\Big]^2, \end{aligned} \quad (17)$$

where

$$\begin{aligned} &s_\pi = (P_{\pi^+} + P_{\pi^-})^2\ ,\ s_\nu\ =\ (P_\nu + P_{\bar{\nu}})^2\ ,\ \sigma_\pi\ =\ (1 - 4m_\pi^2/s_\pi)^{1/2}, \\ &X = \left\{\left[\frac{1}{2}(m_K^2 - s_\pi - s_\nu)\right]^2 - s_\pi s_\nu\right\}^{1/2}. \end{aligned} \quad (18)$$

It leads to a branching ratio

$$Br(K_L \to \pi^+\pi^-\nu\bar{\nu}) = 4.81\cdot 10^{-20}[6a_1 - 6a_3 - 2a_4 + 2\sin^2\theta_W(a_2+2a_4) - \xi]^2\,, \quad (19)$$

which is roughly 10^{-7} smaller compared with the short distance contribution [5].

$$K^+ \to \pi^+\pi^0\nu\bar{\nu}$$

Like the previous decay mode, $K^+ \to \pi^+\pi^0\nu\bar{\nu}$ receives only contribution from the anomaly Lagrangian. The amplitude is given by

$$\begin{aligned} A(K^+ \to \pi^+\pi^0\nu\bar{\nu}) = &-\frac{i\alpha G_8}{4\pi f_\pi M_Z^2 \sin^2\theta_W \cos^2\theta_W}\bar{\nu}\gamma_\mu(1-\gamma_5)\nu \\ &\cdot \left[1 + 3a_3 + a_4 - \sin^2\theta_W(2 - 3a_2 + 6a_3) - \xi\right] \varepsilon^{\mu\nu\alpha\beta} P_{K\nu} P_{\pi^+\alpha} P_{\pi^0\beta}. \end{aligned} \tag{20}$$

The differential decay rate [16,17] is found to be

$$\begin{aligned} \frac{d^3\Gamma}{ds_\pi ds_\nu d\cos\theta_\pi} = &\frac{\alpha^2 G_8^2 \sigma_\pi^3 X^3 \sin^2\theta_\pi s_\pi s_\nu}{2^{15}\pi^7 f_\pi^2 M_Z^4 m_K^3 \sin^4\theta_W \cos^4\theta_W} \\ &\cdot \left[1 + 3a_3 + a_4 - \sin^2\theta_W(2 - 3a_2 + 6a_3) - \xi\right]^2, \end{aligned} \tag{21}$$

where

$$\begin{aligned} s_\pi \;=\; (P_{\pi^+} + P_{\pi^0})^2\;,\; s_\nu \;=\; (P_\nu + P_{\bar{\nu}})^2\;,\; \sigma_\pi \;&=\; (1 - (m_{\pi^+} + m_{\pi^0})^2/s_\pi)^{1/2}, \\ X \;&=\; \left\{\left[\frac{1}{2}(m_K^2 - s_\pi - s_\nu)\right]^2 - s_\pi s_\nu\right\}^{1/2} \end{aligned} \tag{22}$$

The branching ratio is obtained as

$$Br(K^+ \to \pi^+\pi^0\nu\bar{\nu}) = 3.44 \cdot 10^{-19}[1 + 3a_3 + a_4 - \sin^2\theta_W(2 - 3a_2 + 6a_3) - \xi]^2 \tag{23}$$

which is about one order larger than that of $K_L \to \pi^+\pi^-\nu\bar{\nu}$. Since $K^+ \to \pi^+\pi^0\nu\bar{\nu}$ is related to $K_L \to \pi^+\pi^-\nu\bar{\nu}$ by isospin symmetry for both the long and short distance contributions, the relative suppression between the long and short distance effects should be roughly the same.

$$K \to 3\pi\nu\bar{\nu}$$

There are three modes $K_L \to \pi^+\pi^-\pi^0\nu\bar{\nu}$, $K^+ \to \pi^+\pi^0\pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\pi^+\pi^-\nu\bar{\nu}$ receive contributions from the $O(P^2)$ Lagrangian. The processes go through $K \to 3\pi$ and then emit Z^0 from one of the charged meson involved. They are basically internal bremsstrahlung type of processes. Since the leading contribution is of $O(P^2)$, the suppression

is not very strong. Unfortunately, there are no short distance calculations available at the moment because of the smallness of the rate due to the phase space suppression and we only list the decay amplitudes by the long distance contributions for the sake of completeness. They are obtained as follows

$$A(K_L \to \pi^+\pi^-\pi^0\nu\bar{\nu}) = -\frac{i\pi\alpha(1-2\sin^2\theta_W)G_8}{M_Z^2\sin^2\theta_W\cos^2\theta_W}(m_K^2 - 2P_K P_{\pi^0}) \cdot \left[\frac{P^\mu_{\pi^+}}{P_Z(P_Z+2P_{\pi^+})} - \frac{P^\mu_{\pi^-}}{P_Z(P_Z+2P_{\pi^-})}\right]\bar{\nu}\gamma_\mu(1-\gamma_5)\nu \tag{24}$$

$$A(K^+ \to \pi^+\pi^0\pi^0\nu\bar{\nu}) = \frac{i\pi\alpha(1-2\sin^2\theta_W)G_8}{M_Z^2\sin^2\theta_W\cos^2\theta_W}\frac{m_\pi^2 + 2P_{\pi^0}P'_{\pi^0}}{P_Z(P_Z+2P_{\pi^+})}P^\mu_{\pi^+}\bar{\nu}\gamma_\mu(1-\gamma_5)\nu \tag{25}$$

$$\begin{aligned} A(K^+ \to \pi^+\pi^+\pi^-\nu\bar{\nu}) = {} & \frac{i\pi\alpha(1-2\sin^2\theta_W)G_8}{M_Z^2\sin^2\theta_W\cos^2\theta_W} \\ & \cdot\left[\frac{m_K^2 + m_\pi^2 - 2P_K P'_{\pi^+} + 2P'_{\pi^+}P_{\pi^-}}{P_Z(P_Z+2P_{\pi^+})}P^\mu_{\pi^+}\right. \\ & + \frac{m_K^2 + m_\pi^2 - 2P_K P_{\pi^+} + 2P_{\pi^+}P_{\pi^-}}{P_Z(P_Z+2P'_{\pi^+})}P'^\mu_{\pi^+} \\ & \left. - \frac{2m_K^2 - P_K P_{\pi^+} - P_K P'_{\pi^+}}{P_Z(P_Z+2P_{\pi^-})}P^\mu_{\pi^-}\right]\bar{\nu}\gamma_\mu(1-\gamma_5)\nu\,, \end{aligned} \tag{26}$$

where $P'_{\pi^{0(+)}}$ represents the momentum of the second $\pi^0(\pi^+)$ in the relevant mode and P_Z is equal to the difference between the momenta of the kaon and three pions or the sum of the two neutrino momenta.

IV. CONCLUSION

We have calculated the long distance contributions to the decays of $K \to n\pi\nu\bar{\nu}$ within the framework of chiral perturbation theory. For the processes with one or two pions in the final states, the long distance effect is highly suppressed relative to the short distance by a factor of 10^{-7}. We remark that the estimates of the decay rates have been done by the replacement of the divergent loop function of Eq. (12) by its finite part, the so-called chiral logarithmic piece, which is a rather crude approximation. We have also neglected all $O(P^4)$

counter terms in the chiral Lagrangian. Moreover, we have ignored the contribution from the box diagram with two W-boson intermediate states, which might be large as shown in Ref. [2]. Therefore, our results are subject some uncertainties. However, these uncertainties should not change the conclusion that the long-distance contributions to $K \to n\pi\nu\bar{\nu}$ are negligible compared to that from the short-distance parts. Extraction of standard model parameters from these modes suffers no uncertainty from the contamination of the long distance effect.

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